Lesson Outline

This document describes a general lesson outline that can be used to implement any of the Collaborative Mathematics challenge problems in the classroom.

Phase 1: Introduction (approx. 5 minutes)

- Play the chosen challenge video for students. Pause or replay sections as necessary if questions arise about any of the given definitions, the problem setting, or the statement of the challenge.
- Ask students to think individually about the problem for about one minute before sharing their ideas with others in their group. If students are familiar with the idea of *think-pair-share*, this is *think* time.
- While students are thinking, distribute the handout that accompanies the chosen challenge video. Handouts for each of the challenges can be downloaded from the curriculum page of our website.

Phase 2: Exploration (approx. 15 minutes)

- Allow students at least 10 minutes to explore the challenge problem with a small group. Pairs or groups of three are recommended.
- As students are working, circulate among the groups and observe the strategies that they are using to tackle the challenge. Note any especially interesting or creative approaches, even ones that do not lead to a solution. We can often learn from our false starts and unproductive approaches!
- Avoid helping. That is, avoid giving explicit approval of a correct answer, and likewise avoid making explicit corrections of a group with an unproductive or "incorrect" approach. Rather, steer the group using questions like those listed in the box *Guiding Questions for Mathematical Thinking*.
- Students may find some of the challenge problems to be quite difficult. Encourage and praise incremental progress! Partial solutions and insightful observations about individual components of the problem should be held up as valuable steps in the problem-solving process.

Phase 3: Discussion (approx. 15 minutes)

- Prompt students to share their progress, whether or not any group has solved the problem. Emphasize the process over the solution by asking "Where did you start? What were your initial ideas about this problem?" as opposed to asking "What is the answer?"
- As you circulated during the exploration phase, you may have observed certain groups using noteworthy approaches. Ask these groups to share their strategies (even the unproductive ones) as a way to bring out a variety of ideas.
- If no solution has yet been found to the initial challenge, allow students additional time to work in their groups. The discussion phase will likely have spread around the different ideas, so students should now have new insight as they return to group work. After a second round of exploration (5 minutes), have a second round of discussion (5 minutes).

- It may be that students do not arrive at a complete solution within a single class period. In this case, try to avoid simply giving the answer. Instead, encourage students to continue thinking about and discussing the problem. If students keep a mathematical journal, consider assigning a prompt around summarizing their progress to this point.

Guiding Questions for Mathematical Thinking

1. Encourage students to clarify their thinking.
   - Could you explain that further?
   - What do you mean when you say…?
   - Why do you say that?
2. Encourage students to make explicit their assumptions.
   - Is this always the case?
   - Under what circumstances is that the case?
   - Why do you think that assumption holds here?
3. Encourage students to use evidence as a basis for argument.
   - Why do you say that?
   - Can you show an example of what you mean?
   - Is there reason to doubt this evidence?
4. Encourage students to see other points of view.
   - Did anyone see this another way?
   - Is that the only possible explanation (solution, approach, etc.)?
   - What is a counterargument for…?
5. Encourage students to identify relationships.
   - If that happened, what else would result?
   - How does… affect…?
   - What does… imply about…?
6. Encourage students to question their questions.
   - Why do you want to know that?
   - Why is that question important to ask?
   - Why do you think we have been asked this question?

Extensions

The back of each handout includes a few problems that extend the ideas of the main challenge. If time permits, consider a fresh round of exploration and discussion around one of these extensions. Some of the extensions are quite open-ended and may be suitable as journal prompts, or as projects that could be explored over an extended period of time.

Evaluation

When evaluating student performance in a problem-solving experience like this, it is important to place value on the process rather than the solution. Consider using a rubric like the one shown below.

4 Student exhibits excellent creativity/patience/persistence in the problem-solving process, both in written work and verbal participation in small-group and whole-class discussion. All claims are supported by evidence. All explanations are clear and at an appropriate level of detail.

3 Student exhibits good creativity/patience/persistence in the problem-solving process, both in written work and verbal participation. Most, though not all, claims are supported by evidence. Most, though not all, explanations are clear and at an appropriate level of detail.

2 Student exhibits inconsistent creativity/patience/persistence in the problem-solving process, based on written work, verbal participation, or both. Few claims are supported by evidence. Few explanations are given at an appropriate level of detail.

1 Student exhibits little or no creativity/patience/persistence in the problem-solving process, either in written work or verbal participation in small-group or whole-class discussion. Most claims are made without supporting evidence. Most explanations lack sufficient clarity or detail.

0 Makes no effort.